**Instructions for Exercise 11: Dominance Analysis**

**Revised September 2020**

**Preliminary Notes**

1. The training exercise materials are available here: <https://www.york.ac.uk/che/research/equity/handbook/>
2. There are two spreadsheets: “Ex 11 – student file” and “Ex 11 – solution file”. Please open the “student file” to start with and go through the worksheets filling in the relevant cells yourself. You can look at the solution file if you get stuck.
3. When the spreadsheet is first opened a ‘Security Warning’ may be displayed below the menu bar.  Select ‘Enable this content’.
4. This exercise was produced for the Handbook of Distributional Cost-Effectiveness Analysis by Tom Van Ourti and Owen O’Donnell and edited by Richard Cookson with help from Matthias Arnold and Yukiko Asada.

**Introduction**

The aim of this exercise is to learn how to conduct dominance analysis of health distributions, to establish whether one distribution is better than another according to standard ethical principles or “axioms”.

The exercise is based on the illustrative example of Nicotine Replacement Therapy in England. Nicotine replacement therapy (NRT) to help people stop smoking or chewing tobacco is a classic example of a preventive healthcare programme aimed at improving health and reducing health inequality. You are asked to imagine that the UK government is considering three national options for NRT programme delivery in England:

1. No Public NRT: do not provide any public subsidy for nicotine replacement therapy
2. Universal NRT: offer free nicotine replacement therapy to all smokers
3. Proportional Universal NRT: Universal NRT with additional resources to encourage uptake in disadvantaged communities

We provide the group-level distributions of health adjusted life expectancy (HALE) at birth resulting from these three options, based on the calculations detailed in handbook exercises 7, 8 and 9. Your task now is to evaluate which of these three distributions is the best, taking into account both efficiency in terms of sum total health and equity in the distribution of health.

**Getting started**

Open ‘Ex 11 – student.xlsx’ and select the <PD> worksheet.

You must complete the yellow cells.

Table 1 presents the distributions of individual average health-adjusted life expectancy (HALE) resulting from each of the three programmes – i.e. 1: No NRT, 2. Universal NRT and 3. Proportional Universal NRT. Each distribution is broken down by ten social subgroups: five socioeconomic groups within the North and South of England.

Figure 1 presents those distributions in a graph. The differences are too small to be seen in this graph, so we zoom in more closely on those differences in Figure 2. To get a better feel for the population level policy impacts, we also move from the level of individual average HALE to the level of population level total HALYs, by multiplying by population size.

*Pareto dominance (PD)*

Group-level Pareto dominance exists when, as the expected consequence of a decision, at least one group is better off and none are worse off.

i. Complete the yellow cells to fill in Figure 2, a bar chart that shows incremental differences in population total HALYs for programmes 2 and 3 compared with programme 1 (No NRT).

ii. Compare each of the options in terms of group-level Pareto dominance (i.e. whether the option is better for at least one group and not worse for the others).

The Pareto principle indicates higher social welfare under options 2 & 3 compared to option 1, but 2 and 3 cannot be ranked. 3 performs better than 2 among some groups but worse among others.

*First order stochastic dominance (FOSD)*

Open the <FOSD> worksheet.

First order stochastic dominance extends the Pareto principle by adding the anonymity principle that identity does not matter, only consequences. We can thus re-rank the groups in order of health before comparing the distributions of health consequences.

Table 1 repeats our distributions of health-adjusted life expectancy (HALE) at birth for the 10 groups (5 SES groups for both North and South England).

To establish whether there is (inverse) FOSD:

1. Rank the groups from low HALE to high HALE. You should confirm that all three programmes lead to the same ranking of groups.

*Hint: in cell C5, =RANK(E5,E$5:E$14,1)) change Es to Fs and copy down. Then check if the ranks change.*

1. Complete Table 2, which shows the cumulative distribution functions for all three policies against the ranked cumulative population shares.

*Hint: the ranked cumulative population shares are obtained by calculating the cumulative share of the 10 groups (from column group size in Table 1).*

*Hint: Use the VLOOKUP(…;…;…) command.*

1. Look at Figure 1, which shows cumulative distribution functions using a scatter diagram (HALE on the X axis, and fractional health rank on the Y axis).
2. Look at Figure 2, which shows the inverse cumulative distribution functions, for consistency with tests of generalized Lorenz dominance in subsequent worksheets (fractional health rank on the X axis, and HALE on the Y axis).
3. Because individual level differences are small, we cannot see where the inverse cumulative distributions cross. Therefore, we also plot the difference between the inverse cumulative distributions of 3 vs 1 and 2 vs 1 against the fractional health rank in Figure 3.
4. There is inverse First Order Stochastic Dominance of 2 and 3 over 1 (the inverse cumulative distribution functions lie above 1 at all ranks), but the inverse cumulative distribution functions of 2 and 3 cross. This means that the Pareto principle combined with the anonymity principle suffices to assign the lowest level of social welfare to 1, but that 2 and 3 cannot be ranked by this criterion alone.

*Hint: 1 could never FOSD 2 & 3 because mean HALE is lower under programme 1.*

1. See Table 3 for a summary of this information.

*Hint: the additional calculations required for Table 3 are provided in Table 4.*

*Lorenz dominance (LD)*

Select the <LD> worksheet.

To establish a welfare ordering of 2 versus 3, we need to impose additional normative principles. We will examine whether the principle of health transfers (and transitivity of social preferences) by itself suffices to achieve this. According to the health transfers principle, a hypothetical transfer of health from healthier to less healthy people ought to lead to a more equal health outcome. This is a central axiom of all inequality indices, and in the income inequality literature known as the Pigou-Dalton principle. This also means that, for now, we neglect that 2 has a higher mean than 3 (i.e. we assign no normative significance to this difference). Another way to think about this assumption is that one is only concerned about (relative) inequality, which boils down to testing for Lorenz dominance.

1. Complete the line of equality column, which represents with the case where every group has the same share of HALE.

*Hint: the line of equality traces the cumulative population share against itself. It is derived from worksheet <FOSD>*

1. Construct the Lorenz coordinates. These are obtained as the cumulative shares of the three HALE distributions in table 2 of worksheet <FOSD>.

*Hint: Calculate population shares to facilitate the calculation of the cumulative shares.*

1. Look at the Lorenz curves in Figure 1 (cumulative share on Y axis and fractional rank on X axis).
2. To see where the Lorenz curves cross, look at Figure 2 which shows the differences from the Lorenz curve for 1.

*Hint: We consider the differences from 1, and not the more commonly differences from the line of inequality, because the differences between programmes would be too small to see.*

1. We find that relative inequality is lowest in 3 and highest in 1, while 2 is in between. This ranking is achieved by imposing the principle of health transfers and assuming that differences in the means are irrelevant. The latter is obviously a strong assumption.
2. The same information is provided in Table 2.

*Generalized Lorenz dominance (GLD)*

Select the <GLD> worksheet.

While the principle of health transfers suffices to rank 2 and 3 by relative inequality, it does not account for the differences in means and therefore does not establish a social welfare ordering. This can potentially be obtained by checking for generalized Lorenz dominance.

1. In Table 1, derive the generalized Lorenz coordinates. The cumulative population share is obtained from worksheet <FOSD> and the generalized Lorenz coordinates are obtained as the cumulative means of the three HALE distributions in Table 2 of that worksheet.

*Hint: Calculate population shares to facilitate the calculation of the cumulative means.*

1. Graph the GL curves (cumulative mean on Y axis and fractional rank on X axis).
2. To see whether curves cross, plot the difference between the GL curve of a simulated policy and that in the baseline.

*Hint: We consider the differences from 1, and not the more commonly differences from equally distributed health, because the differences between policies would be too small to see.*

1. The generalized Lorenz curves of 2 and 3 cross, indicating that the principle of health transfers (& transitivity) does not suffice to order these distributions in terms of welfare.

*Hint: note that, due to the principle of health transfers, the crossing of the curves occurs at a higher cumulative population share than occurred with the cumulative distributions (compare with Figure 3 in worksheet <FOSD>).*

1. See Table 2 for a summary of this information.

*Second order generalized Lorenz dominance*

Select the <SOGLD> worksheet.

While the principle of health transfers is sufficient to order the three policies in terms of the relative inequality generated, it was insufficient to establish a welfare ordering of 2 and 3. We now examine whether imposing downside positional transfer sensitivity (DPTS) suffices to achieve this by checking for second order generalized Lorenz dominance.

1. In Table 1, derive the SOGL coordinates, which are the cumulative means of the GL coordinates in Table 1 of worksheet <GLD>.

*Hint: the solutions excel sheet provides two equivalent ways of calculation, i.e. as cumulative means of the GL coordinates (3) and as weighted sums of the HALE levels (1 & 2). See also footnote 17 in the chapter.*

1. Graph the SOGL curves and check for SOGLD.
2. To see whether curves cross, look at Figure 2 which plots the difference between the second-order GL curve and that of the baseline distribution.
3. 3 second-order generalized Lorenz dominates 2.
4. See Table 2 for a summary of this information.

If SOGLD were not established, one could proceed to check generalized Lorenz dominance at higher orders.

**Optional extra exercises on concentration curve dominance**

These exercises return to the original simple distribution of five social groups ranked by socioeconomic status, and apply a concentration curve approach to dominance analysis.

*Concentration curve dominance (CD)*

Select the <CD> worksheet.

Consider whether imposing the principle of income-related health transfers (PIRHT) is sufficient to order 2 and 3 by checking for CD, which apart from the ranking by SES is similar to Lorenz dominance. Proceed analogously to what you did with LD.

We find that relative inequality is ranked lowest in 3 and highest in 1, and 2 is in between.

*Generalized concentration curve dominance (GCD)*

While 3 has lower relative inequality than 2, it might well be that PIRHT (in combination with Pareto & transitivity) does not suffice for a welfare ordering. Consider checking for GCD. Select the <GCD> worksheet and proceed as you did to check for GLD but now rank by SES.

We find that the PIRHT is insufficient to order 2 and 3 in terms of welfare.

*Second order generalized concentration curve dominance (SOGCD)*

Now establish whether imposing bivariate downside positional transfer sensitivity (DPTS) suffices to order 2 and 3 in terms of welfare. This is analogous to checking SOGLD. Select the <SOGCD> worksheet and proceed similar to what you did with SOGLD by rank by SES.

We find that 3 second-order generalized concentration curve dominates 2.

*Individual versus between group analysis*

This exercise has neglected variation in HALE within groups. This might be important, particularly when the average health benefit per group is similar across interventions but the within group health inequality differs. However, most studies simulate group level health distributions and so nothing is known about within group inequality (see chapter 3 for further discussion).